

Biaxial Constitutive Equation Development

Eric H. Jordan and Kevin P. Walker
University of Connecticut and Engineering Science Software, Inc.

INTRODUCTION

Some current production gas turbine engines utilize large scale single crystal superalloy hot section components. Structural analysis of these components requires a valid stress-strain-temperature relation, often referred to as a constitutive equation. At present this behavior can only be represented by elastic constitutive equations or pure creep equations. The actual application involves viscoplastic strain cycles that are not represented well by either of these types of equations. The goal of the work described in this report is to develop and verify viscoplastic constitutive equations for superalloy single crystals and directionally solidified alloys.

In developing the constitutive equations an interdisciplinary approach is being pursued. Specifically, both metallurgical and continuum mechanics considerations are recognized in the formulation. Experiments will be utilized to both explore general qualitative features of the material behavior that need to be modeled and to provide a means of assessing the validity of the equations being developed. The model under development explicitly recognizes crystallographic slip on the individual slip systems. This makes possible direct representation of specific slip system phenomena previously studied by metallurgists.

GENERAL FEATURES OF THE PROPOSED MODEL

Viscoplastic constitutive equations currently used for describing the high temperature structural behavior of isotropic superalloys make use of a state variable concept. These viscoplastic equations are basically rate-dependent creep equations in which the creep or inelastic strain rate depends not only of the state of stress, but also on two state variables. These state variables represent the effect of prior inelastic deformation history on the current creep response of the material.

Constitutive equations for describing the anisotropic creep behavior of superalloy single crystals were developed by Paslay, Wells and Leverant [1] in 1971. The present constitutive formulation takes the anisotropic creep theory developed in Reference [1] and incorporates two state variables into the model to account for the effect of prior inelastic deformation history on the current rate-dependent response of the material. This is done in a manner analogous to the isotropic development of viscoplastic state variable models.

SIMULATIONS WITH THE MODEL

Figure 1 shows a numerical simulation of a single crystal superalloy which can exhibit slip on its four octahedral 111 planes in twelve 110 type directions. The single crystal bar specimen is pulled along its axis under load control with the direction of the load axis inclined at various angles to the crystallographic axes. Curve 1 shows the response obtained by loading the bar specimen along its axis under fully reversed load control to a value of ± 207 MPa (± 30 Ksi) at a constant load rate magnitude of 6.9 MPa (1 Ksi) per second. Curves 2 to 6 show the effect of inclining the specimen axis and corresponding load direction by angles of $0 = 9^\circ$, 18° , 27° , 36° and 45° to the crystallographic axes in a plane containing two of the crystallographic axes. The increasing angles correspond to moving along the $[001] - [011]$ side of the stereographic triangle. Curve 7 corresponds

to the $[\bar{1}11]$ vertex of the stereographic triangle.

According to Schmid's law [2] the yield stress decreases from its maximum values at the $[001]$ and $[011]$ vertices of the stereographic triangle to a minimum value at $\theta=22.5^\circ$ halfway between the $[001]$ and $[011]$ vertices. The increased plasticity in the loop for curves 3 and 4 corresponding to $\theta=18^\circ$ and $\theta=27^\circ$ reflect this minimum behavior. Symmetry between the loops equidistant from the minimum at $\theta=22.5^\circ$ is not achieved with the present formulation because Young's modulus for the single crystal bar specimen increases from the $[001]$ vertex to the $[011]$ vertex and from the $[011]$ vertex to the $[\bar{1}11]$ vertex of the stereographic triangle. This increasing value of Young's modulus is reflected in the increasing slope of the curves labeled 1 to 7 in Figure 1.

Figure 2 shows the effect of strain rate on a bar specimen oriented along the $[001]$ direction corresponding to one of the crystallographic axes. The specimen was pulled along its axis under strain control at constant strain rate magnitudes of 10^{-3} to 10^{-6} per second and shows the usual type of positive strain rate sensitivity in which the "yield" stress increases with increasing strain rate.

Octahedral slip on the four 111 planes in the twelve $[112]$ slip directions and cube slip along the planes containing the crystallographic axes are also being coded into the FORTRAN subroutine to provide a constitutive model which can exhibit different types of slip behavior in different temperature ranges.

CONSIDERATIONS RELATING THEORY AND EXPERIMENT

In experiments on isotropic materials involving hollow tube biaxial specimens it is relatively straight forward to calculate from the measured load and torque values using "strength of materials" formulae. Unfortunately, these simple formulae cannot, in general, be used with anisotropic materials. Instead, the specimen must be analyzed as a structure. It is essential to make the analysis of the specimen computationally efficient if simulation of the measured load-displacement histories is to be practical. The approach taken in this investigation is to solve for the stress distribution in the specimen using a specially constructed finite element program written in FORTRAN on the IBM PC-XT computer. This program represents the tubular specimen as a single high order finite element whose shape function includes terms which reflect the spatial symmetry corresponding to the cubic symmetry of the superalloy single crystal. The code for this finite element model has been written and is currently undergoing numerical experiments.

Experiments performed on single crystal superalloys [3-5] have shown that there is a tension-compression asymmetry in the flow stress. This has been accounted for by metallurgists by assuming that the applied stress constricts the Shockley partials during cross slip. Asymmetric flow stress behavior may be incorporated into the model in many ways. The specific manner chosen to incorporate this asymmetry will be chosen in accordance with the constriction stress explanation put forth by metallurgists. Specifically, the drag stress state variable in the constitutive model will be assumed to depend on the constriction stress component.

PROPOSED EXPERIMENTAL PROGRAM

Biaxial experiments with tubular specimens will be conducted at 1600°F on a combined tension-torsion machine. The biaxial setup used to conduct these experiments was developed previously with partial support from NASA grant NAG-3-160. The setup

includes a servo-hydraulic tension-torsion machine and a capacitance probe based extensometer shown in Figure 3. This extensometer utilizes four high temperature non-contacting capacitance probes that do not require cooling. Two probes per channel are used to cancel out cross-talk between extension and rotation. In order to heat the specimen to 1600°F an audio frequency heater has been purchased and is undergoing operation trials.

Gripping a tubular tension-torsion specimen at high temperatures is problematical. To accomplish this the single crystal tube has Hastelloy-X extensions which are vacuum brased onto the specimen. These extensions are welded into disposable 304 stainless steel flanges that bolt to the tension-torsion machine. This procedure eliminates both the need to cool the grips and the need to machine complicated end shapes on the single crystals. The first three specimens have recently been received and are ready for testing.

The testing machine is controlled by a microcomputer. The microcomputer also places the load-displacement results from the tension-torsion machine onto a diskette which can be read by the IBM PC-XT computer and compared with the theoretical predictions from the constitutive model without manual manipulation of the experimental data. This is necessary due to the large volume of data generated in multiaxial testing. In controlling the test machine the digital-to-analog converter sends voltages to a special, highly stable, analog integrator that generates straight line voltage time segments. Without the integrator the command signal would only be stepwise continuous and would involve undefined strain rates between the steps.

The test program is divided into two broad classes of experiments. Specific features of the response which need modeling will be addressed in the first class of experiments, while the second class of experiments is designed to produce target verification tasks for the model. Early experiments will investigate the possibility of removing the effects of prior cycling by stress-free high temperature hold times or by steady cycling. If this can be demonstrated many independent experiments can be performed on one specimen.

Two types of experiments can be performed to assess the values of the two state variables comprising the constitutive model. The value of the scalar drag stress state variable can be determined from experiments in which the strain rate is changed very rapidly, since the proposed equations are such that the stress increment or decrement due to a sudden change in strain rate depends only on the drag stress and not on the tensorial equilibrium stress state variable. To study the value of the tensorial equilibrium stress state variable, creep tests can be performed at various points on a steady state hysteresis loop, searching for the load that gives zero initial creep rate corresponding to the equilibrium stress value. This procedure was used in Reference [6], while the tests involving sudden changes in strain rate have previously been used by Krempl in Reference [7].

REFERENCES

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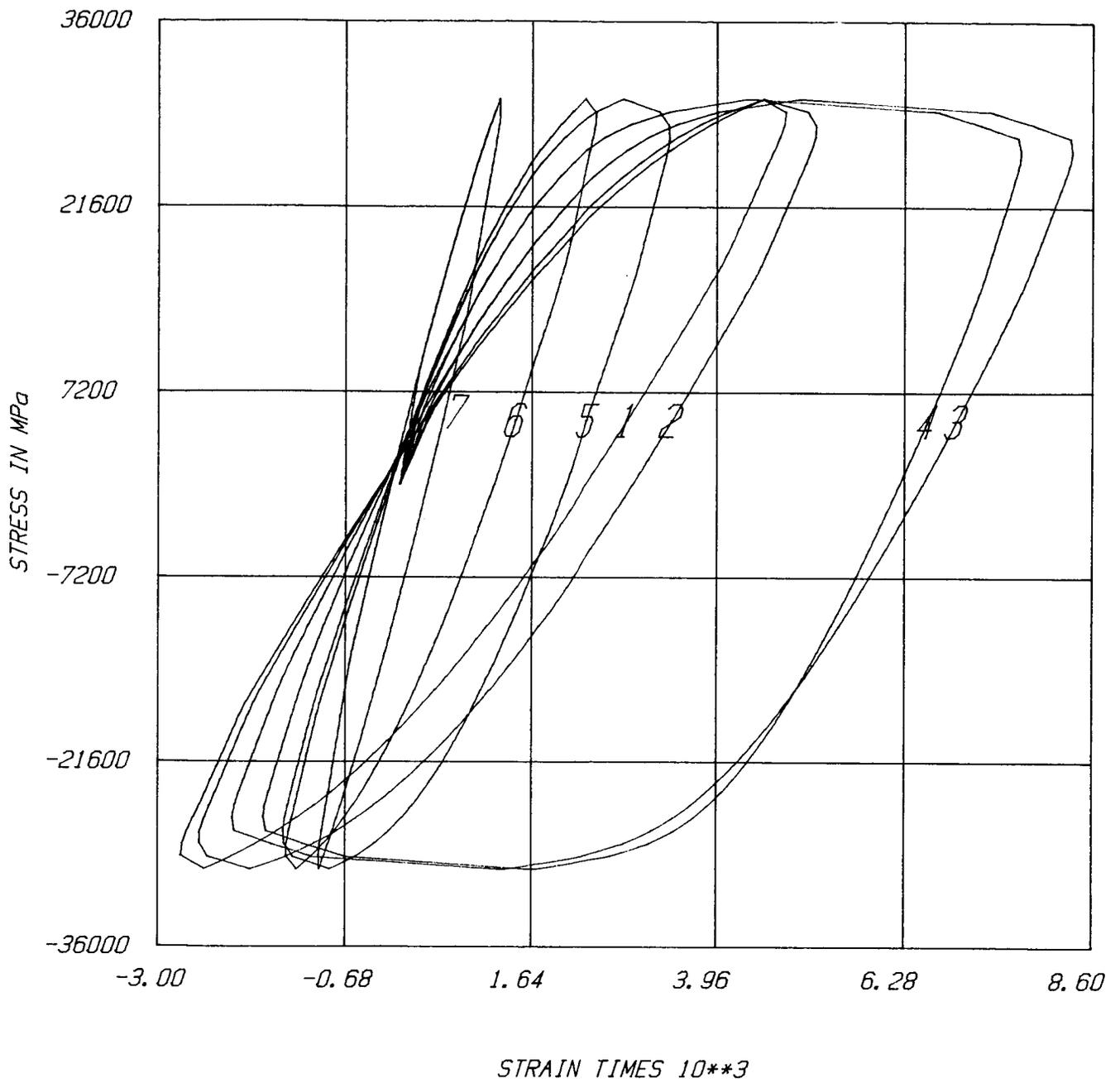


Figure 1. Orientation Dependence of Hysteresis Loops

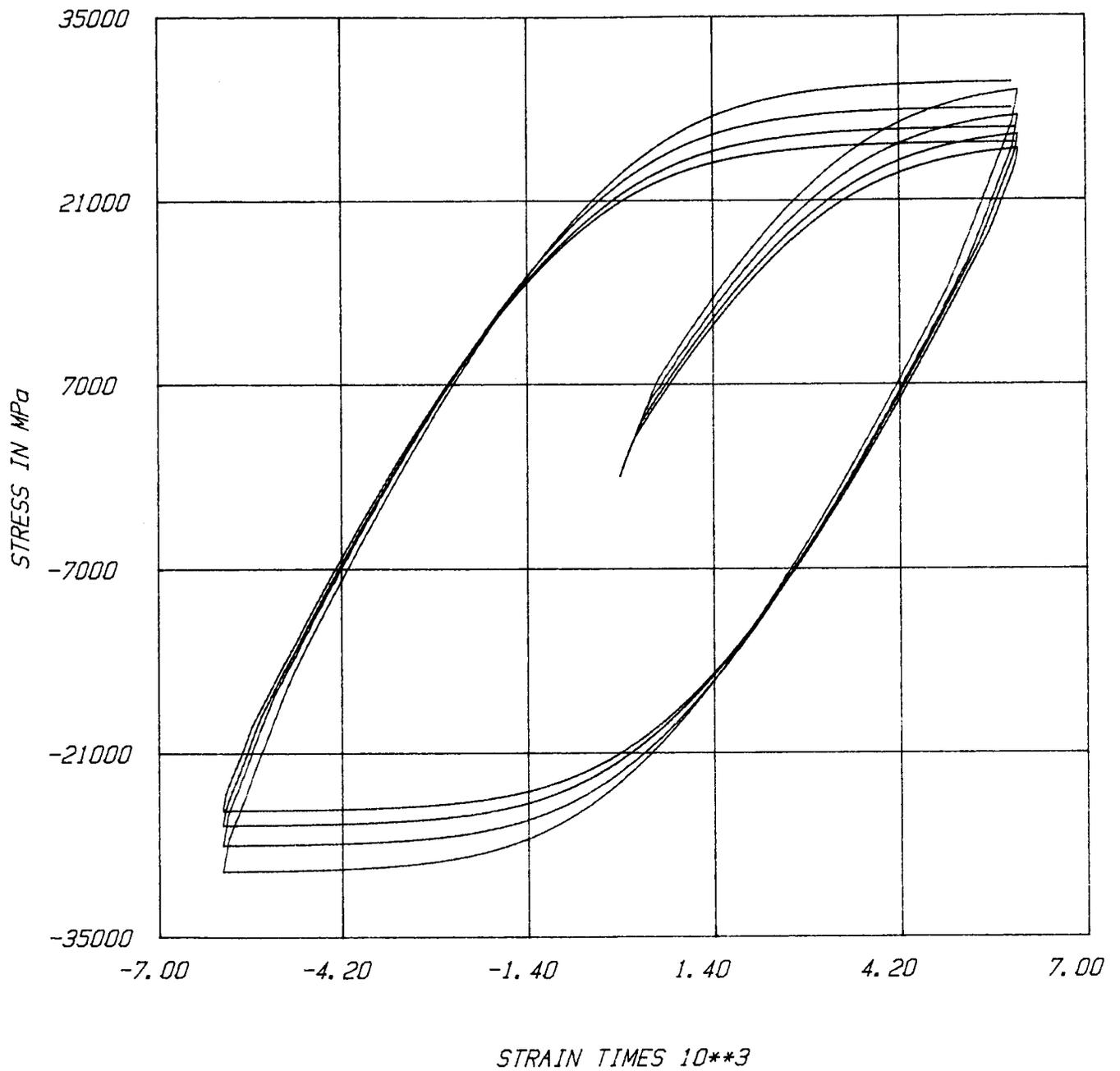


Figure 2. Strain Rate Dependence of Hysteresis Loops

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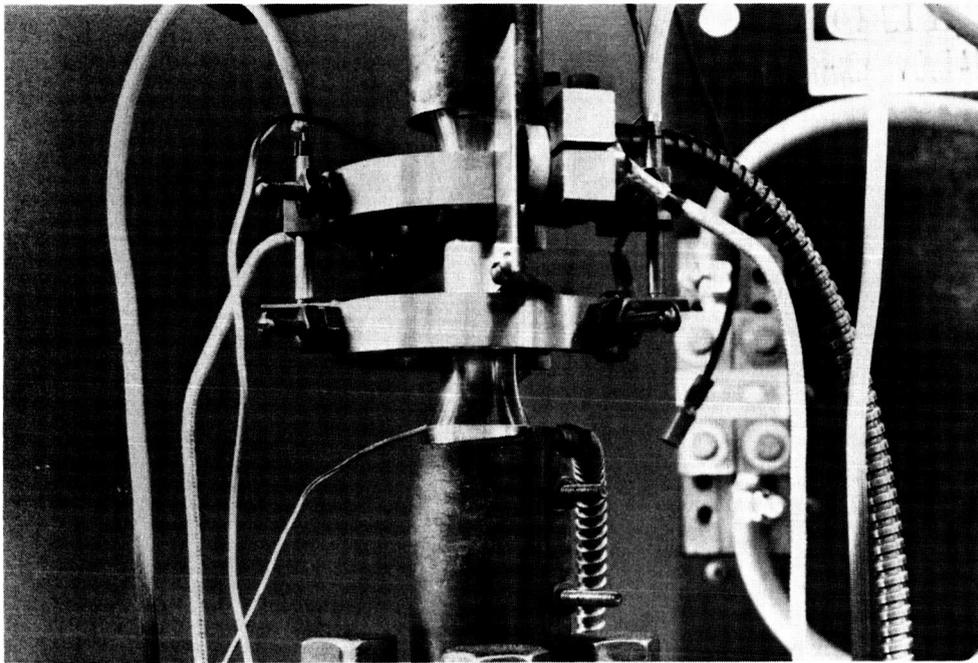


Figure 3: Capacitance Probe Based High Temperature Biaxial Extensometer